Function of two or more Variables:—

Let x, y be two independent Variables If a Variable z takes a Value Corresponding to a pair of Values (x, y) then we say z is a function of two Variables x, y and

we write $z = F(x, y)$

e.g $z = x^2 + xy$ is a function two Variable

The above definition Can be extended for more than two Variables.

Partial differentiation :—

A partial differentiation of a function of two or more Variables is the ordinary differention w.r to one of the Variable when all the remaining Variables are Kept Constants

Let $u = u(x, y)$ be a Function two Variables

Hence u is dependent Variables & x, y are independent.

The partial differentiation of $u = u(x, y)$ w.r to x is the ordinary derivative of u w.r to x. Keeping y is Constant &

it is denoted by

$\frac{\partial u}{\partial x}$, u_x and is Known as.

First order Partial derivative of u w.r.to x and is defined as

$$\frac{\partial u}{\partial x} = \lim_{\Delta x \to 0} \frac{u(x+\Delta x, y) - u(x,y)}{\Delta x}$$

Similarly Partial derivative of u w.r to y is the ordinary derivative of u w.r.to y keeping x is Constant is denoted defined by

$$\frac{\partial u}{\partial y} = \lim_{\Delta y \to 0} \frac{u(x, y+\Delta y) - u(x,y)}{\Delta y}$$ if this

Similarly $\frac{\partial^2 u}{\partial x^2}, \frac{\partial^2 u}{\partial y^2}, \frac{\partial^2 y}{\partial x \partial x}$ are known.

as Second order Partial derivation of w.r.to x, y defined by

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right)$$

$$= \lim_{\Delta x \to 0} \frac{u_x(x+\Delta x, y) - u_x(x,y)}{\Delta x}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial u}\left(\frac{\partial u}{\partial y}\right) = \lim_{\Delta y \to 0} \frac{u_y(x, y+\Delta y) - u(x,y)}{\Delta y}$$

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right)$$